

Proton and neutron charge form factors in soliton model
with dilaton-quarkonium field.

LASSILA K.E.¹

International Institute of Theoretical and Applied Physics, Ames, USA

MAGAR E.N.²

*Laboratory of Computing Techniques and Automation,
Joint Institute for Nuclear Research, Dubna, Russia*

NIKOLAEV V.A.³

*Science Applications and Human Resources Agency,
London, UK*

NOVOZHILOV V.YU.⁴

S-t Petersburg State University, St.Petersburg, Russia

TKACHEV O.G.⁵

*Institute of Physics and Information Technologies,
Far East State University, Vladivostok, Russia*

E-mail: ¹*klassila@iastate.edu* ²*magar@hepalpha1.physics.smu.edu*

³*chiralnet@myfreeoffice.com* ⁴*novozhil@heps.phys.spbu.ru* ⁵*tkachev@ifit.phys.dvgu.ru*

Abstract

Nucleon electromagnetic form factors are considered in the framework of the generalized Skyrme model with dilaton-quarkonium field. In our recent publication we have got big discrepancies between calculated form factors and dipole approximation formula. Here we have reasonably good accordance between them in finite impulse region after vector meson dominance have been taken into account. Omega and Rho-meson have been included into only hadron structure of the photon.

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1 Introduction

The Lagrangian appropriate for a generalized Skyrme model in the leading classical field approximation yields chiral soliton solutions. These solitons are associated with the nucleon and will be used for obtaining spatial structure information on baryons. Effectively, then, the baryon or nucleon form factors can be extracted from the soliton model equations of motion. Experimentally measured form factors can be compared with these soliton predicted spatial densities to test the model.

The present paper includes discussion of our recent work, extensions, and calculations of the nucleon electromagnetic form factors in the generalized Skyrme model. This model involves the theoretical description of the dilaton-quarkonium scalar field and shows its importance in the description of soliton dynamics. We use "dilaton-quarkonium" scalar field to indicate the way we subdivide the gluon condensate in the calculation. This generalized model reproduces the experimental value of the nucleon mass, the input being the experimental value of the pion decay constant and the theoretically derived value of the Skyrme constant, $e = 2\pi$. [1]. The naive, straightforward calculation of the electromagnetic form factors has shortcomings: the values of F_π and e give too small a nucleon size and the calculated curves do not give the approximate dipole form factor values.

The generalized Skyrme model under consideration follows the formulation of Andrianov *et al.* [2] This approach uses the framework of the joint chiral and conformal bosonization of the QCD Lagrangian, including chiral and scalar dilaton-quarkonium fields. In such a model the properties of the topological solitons are dramatically changed in numerical value from those in the original Skyrme model. Several authors have introduced an additional scalar field to the Skyrme model for different motivational reasons. For example, Riska and Schwesinger [3] appear to be the first to investigate the isospin independent part of the nucleon-nucleon spin-orbit interaction when a scalar field is added. A number of papers studied the effects a scalar σ meson would have by introducing it as a gluon condensate. [4], [5], [6], and also related, [2] and [7]. The purely theoretical and convincing reason is that with the introduction of a scalar field, the conformal anomaly, one of the distinctive features of the QCD Lagrangian, is reproduced. In the $SU(2)$ sector of this Lagrangian, one can construct an effective theory which reproduces the conformal anomaly in the framework of the effective Lagrangian method, introducing a field corresponding to scale invariance. As shown in [8], [9], and [10], it leads to the necessary strong attraction at intermediate internucleon distances. In such an approach the starting point is the fermion integral over quark fields, in the low energy regime of QCD. The integral is specified by the finite mode regularization scheme with a cut-off that also plays the role of a low energy boundary. Performing the joint chiral and conformal bosonization on this integral leads to an effective action for chiral $U(x)$ and dilaton $\sigma(x)$ fields. This Lagrangian favors the linear sigma model in terms of the composite field $U(x)\exp(-\sigma(x))$. The resulting effective Lagrangian [2], generalizing the original Skyrme Lagrangian is

$$L_{eff}(U, \sigma) = \frac{F_\pi^2}{4} \exp(-2\sigma) \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] +$$

$$\begin{aligned}
& \frac{N_f F_\pi^2}{4} (\partial_\mu \sigma)^2 \exp(-2\sigma) + \\
& + \frac{1}{128\pi^2} \text{Tr}[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2 - \\
& \frac{C_g N_f}{48} (e^{-4\sigma} - 1 + \frac{4}{\varepsilon} (1 - e^{-\varepsilon\sigma}))
\end{aligned} \tag{1}$$

where the pion decay constant is taken as the experimental value, $F_\pi = 93 \text{ MeV}$ and N_f is the number of flavors. The gluon condensate, according to QCD sum rules, is $C_g = (300 - 400 \text{ MeV})^4$ [11]. The first two terms are the kinetic terms for the chiral and scalar fields and the third term, the well-known Skyrme term. The effective potential for the scalar field is the result of an extrapolation [2] of the low energy potential to high energies by use of a one-loop-approximation to the Gell-Mann Low QCD β - function. The parameter ε is determined by the number of flavors N_f as $\varepsilon = 8N_f/(33 - 2N_f)$.

2 The Nucleon

In the baryon sector we choose the chiral field as the spherically symmetric ansatz of Skyrme and Witten, $U(\vec{x}) = \exp[-i\vec{\tau}\vec{n}F(r)]$, where $\vec{n} = \vec{r}/|\vec{r}|$. It is convenient to introduce a new field, $\rho(x) = \exp(-\sigma(x))$. Then, the mass functional in dimensionless variables, $x = eF_\pi r$, has the form $M = M_2 + M_4 + V$, where

$$M_2 = 4\pi \frac{F_\pi}{e} \int_0^{+\infty} dx \left[\frac{N_f}{4} x^2 (\rho')^2 + \rho^2 \left(\frac{x^2 (F')^2}{2} + \sin^2 F \right) \right], \tag{2}$$

$$M_4 = 4\pi \frac{F_\pi}{e} \int_0^{+\infty} dx \left(\frac{\sin^2 F}{2x^2} + (F')^2 \right) \sin^2 F, \tag{3}$$

$$V = 4\pi \frac{F_\pi}{e} D_{eff} \int_0^{+\infty} dx x^2 [\rho^4 - 1 + \frac{4}{\varepsilon} \cdot (1 - \rho^\varepsilon)]. \tag{4}$$

In the last equations, the same Skyrme parameter value, $e = 2\pi$, is used. The contribution of the potential to the mass is determined by the factor $D_{eff} = C_g N_f / 48 e^2 F_\pi^4$. The mass functional leads to a system of equations for the profile functions $F(x)$ and $\rho(x)$, where a prime is used to denote the derivative with respect to x ,

$$\begin{aligned}
& F''[\rho^2 x^2 + 2\sin^2 F] + 2F'x[x\rho\rho' + \rho^2] + (F')^2 \cdot \sin(2F) - \\
& - \rho^2 \cdot \sin(2F) - \sin(2F) \cdot \sin^2 F/x^2 = 0,
\end{aligned} \tag{5}$$

$$\frac{N_f}{2} x[x\rho'' + 2\rho'] - 2\rho \left[\frac{x^2 (F')^2}{2} + \sin^2 F \right] -$$

$$-4D_{eff} \cdot [\rho^3 - \rho^{\varepsilon-1}]x^2 = 0 , \quad (6)$$

At small distances, $F = \pi N - \alpha x$ and $\rho = \rho(0) + \beta x^2$, with $\rho(0) \neq 0$. For large x , these functions behave as $F(x) \sim a/x^2$, and $\rho(x) \sim 1 - b/x^6 + \dots$

According to the virial theorem,[12] the contributions of the individual terms of the mass functional to the energy of the system must obey the condition,

$$M_4 - M_2 - 3V = 0 , \quad (7)$$

which can be used to control the accuracy of the numerical solution of the system. There are nontrivial equations between the numbers α and β , a and b ,

$$b = \frac{1}{2}a^2/D_{eff} , \quad (8)$$

$$\beta = \left[\rho(0)\alpha^2 + \frac{4}{3}(\rho^3(0) - 1) D_{eff} \right] / N_f . \quad (9)$$

The choice of boundary conditions ensures a finiteness of the mass functional for a given value of the topological charge $B = N$. Performing canonical quantization of the rotational degrees of freedom with the collective variable method,[13] one obtains for the nucleon mass,

$$M_B = M + S(S+1)/(2I) , \quad (10)$$

where the moment of inertia is

$$I = \frac{8\pi}{3}(F_\pi e)^{-3} \int_0^\infty dx \sin^2[\rho^2 x^2 + (F')^2 x^2 + \sin^2 F] . \quad (11)$$

Some numerical results are presented in Table 1, where the soliton mean square radius of

	Present work	Original Model
M	839 MeV	1098 MeV
$\langle r^2 \rangle^{1/2}$	0,44 Fm	0,42 Fm
M_B	1026 MeV	1288 MeV

Table 1: Static properties ($N_f = 2$) in the generalized Skyrme model with $F_\pi = 93 MeV$, $e = 2\pi$, $C_g = (300 MeV)^4$. Also, the results following from the original Skyrme model are given for comparison purposes.

the corresponding baryon $\langle r_{IS}^2 \rangle$ density distribution is given

$$\langle r_B^2 \rangle^{1/2} = \frac{1}{F_\pi e} \left\{ -\frac{2}{\pi} \int_0^\infty dx x^2 F' \sin^2 F \right\}^{1/2} . \quad (12)$$

A discussion of partial restoration of chiral symmetry in this model is given in Ref.([1]. The restoration appears as a large deviation of $\rho(0)$ from its asymptotic value of $\rho(0) = 1$. The dependence of the mass spectra on the gluon condensate in the generalized Skyrme model was also discussed in [1].

3 Form factors of charge distributions

The nucleon electric and magnetic form factors, $G_E(q^2)$ and $G_M(q^2)$ can be calculated from the electromagnetic currents, in the Breit frame where the photon does not transfer energy.

$$\begin{aligned} \langle N_f(\frac{\vec{q}}{2}) | J_0(0) | N_i(-\frac{\vec{q}}{2}) \rangle &= G_E(\vec{q}^2) \xi_f^+ \xi_i , \\ \langle N_f(\frac{\vec{q}}{2}) | \vec{J}(0) | N_i(-\frac{\vec{q}}{2}) \rangle &= \frac{G_M(\vec{q}^2)}{2M_N} \xi_f^+ i\vec{\sigma} \otimes \vec{q} \xi_i . \end{aligned} \quad (13)$$

Here, $|N(\vec{p})\rangle$ is the nucleon state with momentum \vec{p} , ξ_i , ξ_f and two component Pauli spinors, and $\vec{q} \equiv$ momentum transfer.

The isoscalar (S) and isovector (V) nucleon form factors are related to those for the proton and neutron by

$$G_{E,M}^{p,n} = G_{E,M}^S \pm G_{E,M}^V \quad (14)$$

These form factors are normalized to the respective charge and magnetic moments by

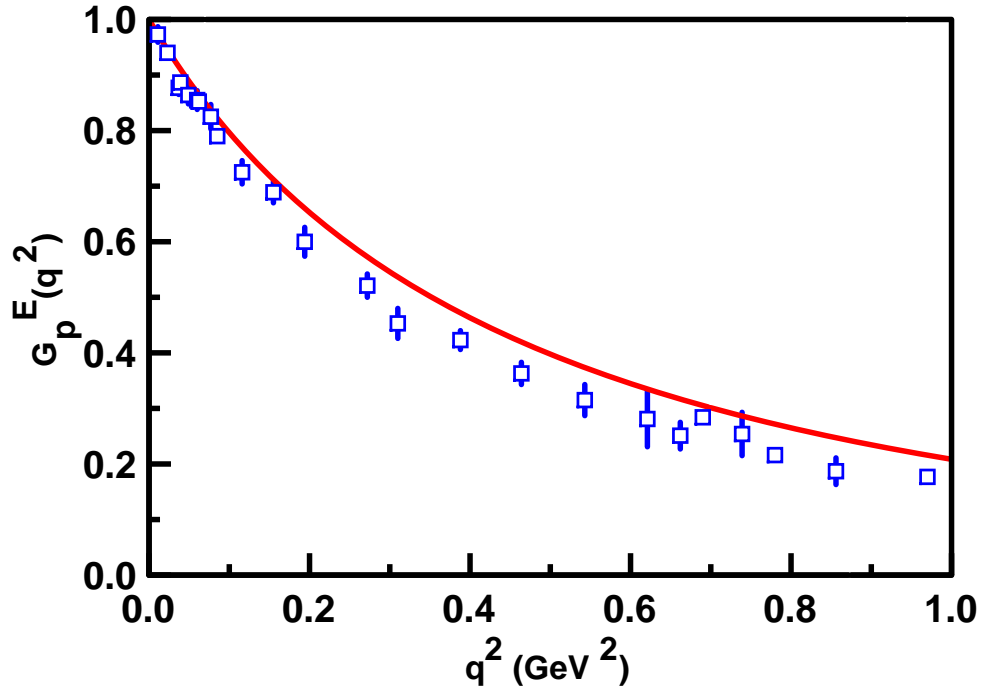


Figure 1: Proton electric form factor as a function of q^2 in GeV^2 calculated for $F_\pi = 93$, $e = 2\pi$, $N_f = 2$, $C_g = (300 MeV)^4$, and $m_\pi = 139$. The experimental data shown come from Ref. [14]

$$\begin{aligned} G_E^p(0) &= 1 & G_E^n(0) &= 0 \\ G_M^p(0) &\equiv \mu_p = 2.79 & G_M^n &= \mu_n = -1.91 . \end{aligned} \quad (15)$$

We remarked above on the smallness of the nucleon size as determined by the baryon charge density distribution in the model with a dilaton-quarkonium field.

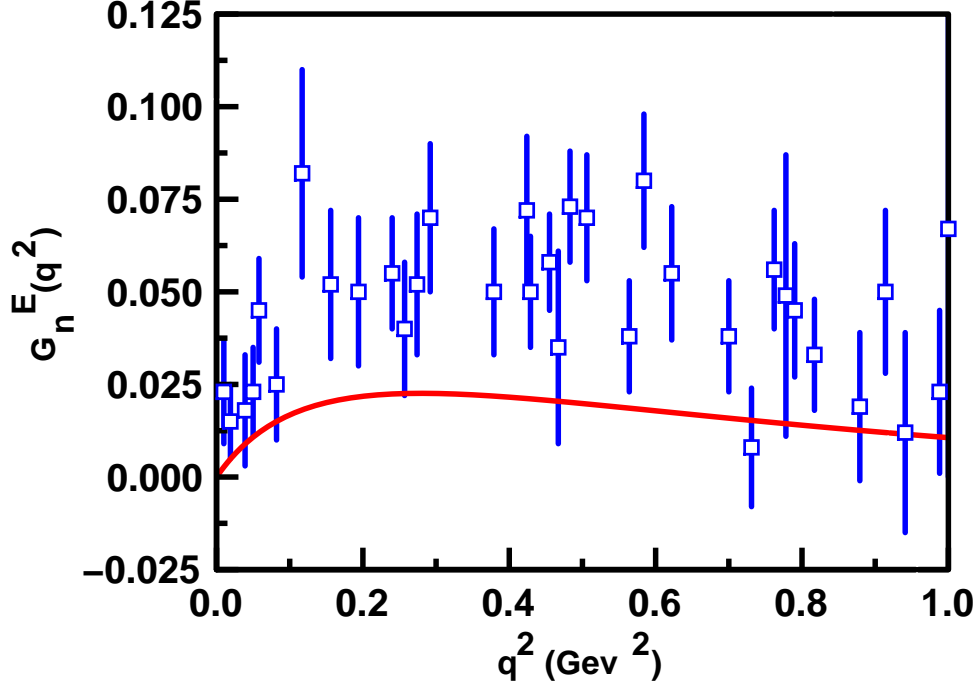


Figure 2: Neutron electric form factor as a function of q^2 in GeV^2 calculated for $F_\pi = 93$, $e = 2\pi$, $N_f = 2$, $C_g = (300MeV)^4$, and $m_\pi = 139$. The experimental data shown come from Ref. [14].

Vector meson dominance means that the isoscalar photon sees ω meson *structure*, but not the isoscalar baryon density $B_0(r)$.

According to vector meson dominance, the isoscalar current is proportional to the ω_μ -field,

$$J_{I=0}^\mu = -\frac{m_\omega^2}{3g} \omega_\mu(r) \quad (16)$$

and the corresponding charge form factor,

$$G_E^S(\vec{q}^2) = -\frac{m_\omega^2}{3g} \int d^3r \exp i\vec{q}\vec{r} \omega(r) . \quad (17)$$

The static $\omega(r)$ obeys the equation,

$$(\nabla^2 - m_\omega^2)\omega(r) = \frac{3g}{2}B(r) = -\frac{3gF'(r)}{4\pi r^2}\sin^2 F(r) . \quad (18)$$

From this equation, we obtain,

$$G_E^S(\vec{q}^2) = -\frac{1}{2}\frac{m_\omega^2}{m_\omega^2 + \vec{q}^2}4\pi \int dr r^2 B_0(r) j_0(qr) . \quad (19)$$

Therefore, the effective isoscalar nucleon density is equal baryon charge density $B_0(r)$ times the ω -meson propagator.

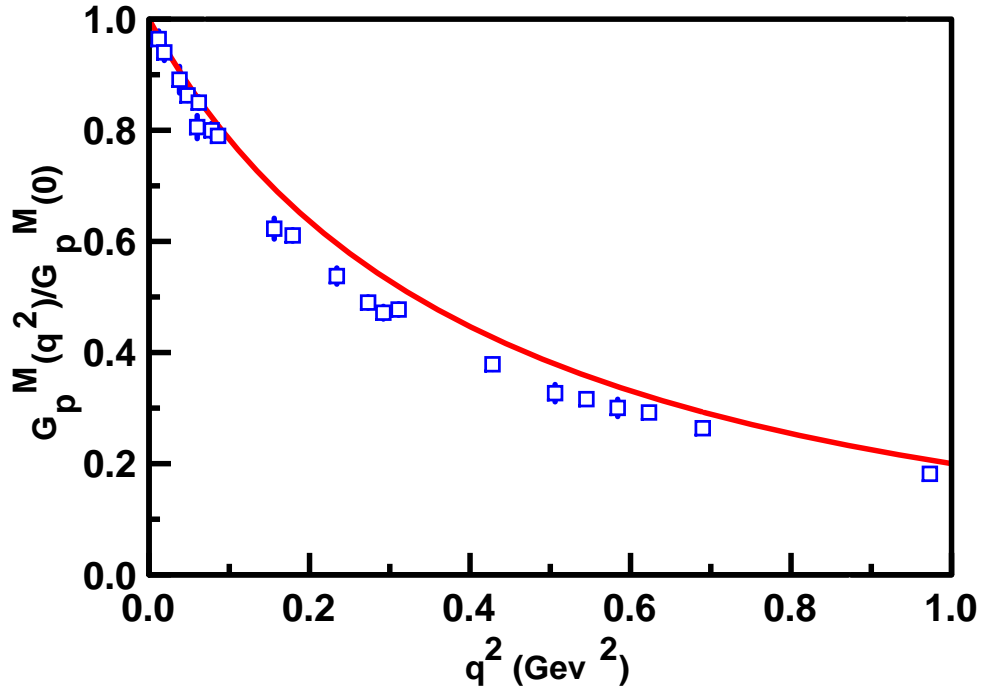


Figure 3: Proton magnetic form factor as a function of q^2 in GeV^2 calculated for $F_\pi = 93$, $e = 2\pi$, $N_f = 2$, $C_g = (300MeV)^4$, and $m_\pi = 139$. The experimental data shown come from Ref. [14].

The isovector electromagnetic formfactor has analogous structure,

$$G_E^V(\vec{q}^2) = -\frac{1}{2}\frac{m_\rho^2}{m_\rho^2 + \vec{q}^2}F_E^V(q^2) , \quad (20)$$

In writing the propagators separately, as a factor multiplied into F_E^V , ω and ρ , themselves have no substructure or internal dynamics; the corresponding Skyrmin densities are

considered as the sources of these ω and ρ fields. Explicit considerations of the role of vector mesons in the electromagnetic form factors in the σ model has been given by Holzwarth[15] and quantum corrections to the relevant baryon properties in the chiral soliton models has been calculated.[16]

The results of the present calculations are given in Figures 1 to 4. The isoscalar part

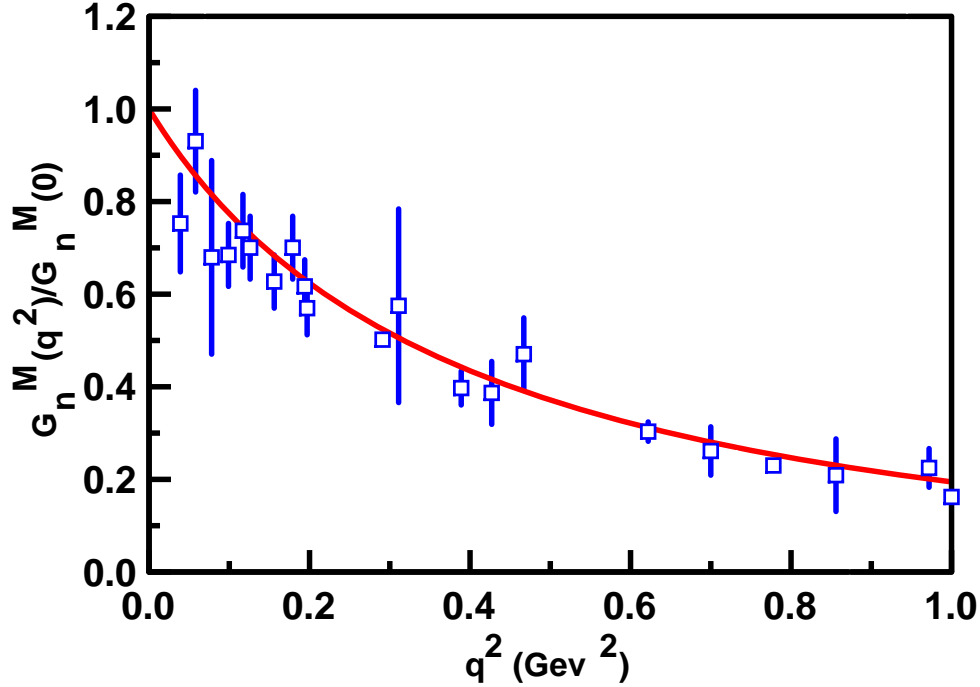


Figure 4: Neutron magnetic form factor as a function of q^2 in GeV^2 calculated for $F_\pi = 93$, $e = 2\pi$, $N_f = 2$, $C_g = (300MeV)^4$, and $m_\pi = 139$. The experimental data shown come from Ref. [14]

of the Skyrmon electric charge coincides with the baryon density distribution, and for the isovector density from the Skyrmon model one obtains,

$$\rho^V(x) = \sin^2 F(x) \left[x^2 \rho^2(x) + (F'(x))^2 x^2 + \sin^2 F(x) \right] . \quad (21)$$

To take chiral symmetry breaking into account, we must add the pion mass term,

$$\mathcal{L}_\pi = \frac{1}{4} m_\pi^2 F_\pi^2 e^{-3\sigma} \text{Tr} \left[U + U^\dagger - \frac{3}{2} e^{-\sigma} \right] , \quad (22)$$

to our Skyrme model Lagrangian. The theoretical predictions for the proton electric, neutron electric, proton magnetic and neutron magnetic form factors, compared with data are shown in Figs. 1, 2, 3, and 4, respectively. Corresponding values of the proton and neutron mean square radius of the electric charge distribution are $0.78 \text{ } Fm^2$ and $-0.19 \text{ } Fm^2$.

4 Conclusions

We have presented our calculations on the nucleon electromagnetic form factors in the framework of the generalized Skyrme model with dilaton quarkonium field. The first calculation [1] in such a model yielded large deviations of the calculated form factors from the dipole approximation formula. In the present work, we use the empirical value of the pion decay constant and the theoretical value for the Skyrme term constant in the vector meson dominance approach to obtain a good description of the form factor data in the finite range of momentum transfer in the measurements. The vector mesons are included only as elements of the hadron substructure of the photon and are not considered as components of the structure in the soliton self-dynamics. Implicit in the approach, though not explicitly proposed, is the possibility of having the role of vector mesons given by higher derivative terms in the effective Lagrangian for soliton dynamics [17]. For example, keeping terms to four orders in the expansion of the effective Lagrangian would lead to a ρ meson-like term and the sixth order terms would give ω -like terms which are important in the calculations of the form factors at the larger momentum transfers.

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